x	$f(x)$
2	12
4	48
6	108
8	192
10	300

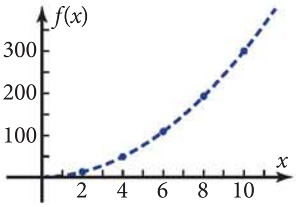

Figure 2-3i

a. Show that the function $f(x) = 3x^2$ fits the data, as shown in Figure 2-3i.

b. Select radian mode, and then plot $f_1(x) = 3x^2$ and $f_2(x) = 3x^2 + 100\sin\left(\frac{\pi}{2}(x)\right)$, where "sin" is the sine function (see Chapter 5). Sketch the result. Does the equation of $f_2(x)$ also fit the given data?

c. Deactivate $f_2(x)$ from part b and plot $f_3(x) = 3x^2\cos(\pi x)$, where "cos" is the cosine function (see Chapter 5). Sketch the result. What do the results tell you about fitting functions to discrete data points?

31. *Incorrect Point Problem:* By considering second differences, show that a quadratic function does *not* fit the values in this table.

x	y
4	5
5	7
6	11
7	17
8	27

What would the last y-value have to be for a quadratic function to fit the values exactly?

32. *Cubic Function Problem:* Figure 2-3j shows the graph of the cubic function

$$f(x) = x^3 - 6x^2 + 5x + 20$$

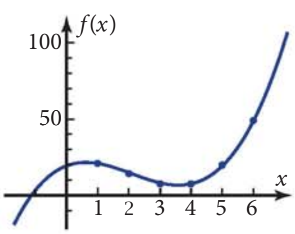

Figure 2-3j

a. Make a table of values of $f(x)$ for each integer value of x from 1 to 6.

b. Show that the *third differences* between the values of $f(x)$ are constant. You can calculate the third differences in a time-efficient way using the list and delta list features of your grapher. If you do it by pencil and paper, be sure to subtract (value − previous value) in each case.

c. Make a conjecture about how you could determine whether a quartic function (fourth degree) fits a set of points.

33. *The Add–Add Property Proof Problem:* Prove that for a linear function, adding a constant to x adds a constant to the corresponding value of $f(x)$. Do this by showing that if $x_2 = x_1 + c$, then $f(x_2)$ equals a constant plus $f(x_1)$. Start by writing the equations of $f(x_1)$ and $f(x_2)$, and then make the appropriate substitutions and algebraic manipulations.

34. *The Multiply–Multiply Property Proof Problem:* Prove that for a power function, multiplying x by a constant multiplies the corresponding value of $f(x)$ by a constant as well. Do this by showing that if $x_2 = cx_1$, then $f(x_2)$ equals a constant times $f(x_1)$. Start by writing the equations of $f(x_1)$ and $f(x_2)$, and then make the appropriate substitutions and algebraic manipulations.

35. *The Add–Multiply Property Proof Problem:* Prove that for an exponential function, adding a constant to x multiplies the corresponding value of $f(x)$ by a constant. Do this by showing that if $x_2 = c + x_1$, then $f(x_2)$ equals a constant times $f(x_1)$. Start by writing the equations of $f(x_1)$ and $f(x_2)$, and then make the appropriate substitutions and algebraic manipulations.

36. *The Constant-Second-Differences Property Proof Problem:* Let $f(x) = ax^2 + bx + c$. Let d be the constant difference between successive x-values. Find $f(x + d)$, $f(x + 2d)$, and $f(x + 3d)$. Simplify. By subtracting consecutive $f(x)$-values, find the three first differences. By subtracting consecutive first differences, show that the two second differences equal the constant $2ad^2$.